single and double pickup cross sections σ_{21} and σ_{20} were also made for He^{++} on the targets He and H_2. Like σ_+ and σ_-, these total charge-changing cross sections may contain contributions from several types of elementary collision events, in which the unobserved target molecule may be left in various charge states, with the release of various numbers of free electrons. However, provided only that double-electron pick up by fast He^+ ions may be neglected, it is nevertheless true for the case of He^+ that the difference $(\sigma_+-\sigma_-)$ from our measurements should be equal to the difference $(\sigma_{10}-\sigma_{12})$; similarly for the case of He^{++}, our $(\sigma_+-\sigma_-)$ should be strictly equal to the sum $(\sigma_{21}+\sigma_{20})$ of the pick-up cross sections. Figure 11 shows this comparison for one of the six cases for which both sets of cross sections are available, that of He^+ in He. There is excellent agreement at low energies, but about a factor of two difference at the high-energy end. However, it may be noted from Fig. 3 that our σ_+ and σ_- differ by only about 10% in this region, so that the discrepancy implies no errors worse than our quoted 5% relative error for σ_+ and σ_-. [By contrast, σ_{12} is an order of magnitude greater than σ_{10} in this region; therefore it is quite possible that the $(\sigma_{10}-\sigma_{12})$ difference is more accurate than our $(\sigma_+-\sigma_-)$ difference.]

Similar comparisons for the other five cases will not be shown here. Actually, the selected case is one of the least favorable. The comparison for He^+ in H_2 is quite similar, but the comparisons for He^+ in Ar and N_2, and for He^{++} in He and H_2 all give generally excellent agreement throughout the energy range of our measurements.

Because our ion production measurements and the total charge-changing measurements are conducted in entirely different ways that are subject to quite different kinds of possible systematic errors, the rather good cross correlations obtained are regarded as gratifying confirmation of both sets of measurements.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the assistance of J. W. Martin, who operated the Van de Graaff accelerator for the bulk of these measurements, and of L. J. Puckett, who assisted with the He^{++} measurements.

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Cross Sections for Ion and Electron Production in Gases by Fast Helium Ions (0.133–1.0 MeV). II. Comparison with Theory*

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The theory of ionization of gases by fast, but nonrelativistic, ions is briefly reviewed, with emphasis on the correspondences expected at high energies among the cross sections for various projectile ions incident on a given target gas. The problem of deducing cross sections for simple ionization from experimental gross ion and electron production cross sections is discussed. The methods developed are applied to the experimental data presented in the preceding paper. Estimates are thus obtained of the simple ionization cross sections for He^+ ions incident on H_2, He, Ar, and N_2 in the energy range 0.133 to 1.0 MeV, and He^+ on Ne, O_2, and CO in the energy range 0.6 to 1.0 MeV. Similar results are obtained for He^{++} ions incident on H_2 and He in the energy range 0.5 to 1.0 MeV. These results are compared with semitheoretical predictions based upon empirical values of target-gas matrix elements determined from previous measurements of the ionization cross sections for protons incident on the same targets. It is shown that the ionization cross sections for He^{++} conform rather well to the predictions, and that the ionization cross sections for He^+ ions are in good agreement with those expected for a point-charge ion with the helium mass and an "effective charge" of $Z'e=+1.2e$. The ionization cross sections are also compared with explicit detailed calculations in the full Born approximation for the cases where such calculations are available. The agreement obtained is quite good in general.

I. INTRODUCTION

In the paper immediately preceding (I. Experimental[1]) we reported apparent cross sections for production of positive ions, σ_+, and free electrons, σ_-, by He^+ ions incident on He, Ne, Ar, H_2, N_2, O_2, and CO and by He^{++} ions on He and H_2. The He^+ measurements covered the energy range 0.133 to 1.0 MeV; the energy of the He^{++} projectiles was varied over the range 0.5 to 1.0 MeV. These cross sections include contributions from any charge-changing events (in which the projectile gains or loses electrons) and from simple ionization events (in which one or more electrons are ejected from the target molecules without a change in the charge state of the incident particle). Information on the total cross sections for charge-changing collisions

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is available from other types of experiments for fast hydrogen and helium ions in some of the target gases studied here.[2–4] For these cases it is possible to make a reliable estimate of the contribution of charge-changing processes to the total ion and electron production, so that the simple ionization contribution can be obtained by subtraction. Less reliable estimates can also be made for the remaining cases. The resulting quantity is called the *apparent* ionization cross sectoin (σ_i) because it contains weighted contributions from events in which multiple electron ejection from the target occurs. In the case of ionization by electron impact, extensive experiments have shown that only a single electron is ejected in about 90% of the ionization events, and similar results have been found in heavy-particle collision studies.[5] Consequently, these apparent ionization cross sections may be profitably compared with theoretical calculations on single electron ionization, as is done in this paper. Because of the hybrid nature of the cross sections for total positive-ion and electron production, it is not possible to make direct comparisons of σ_+ and σ_- with theoretical predictions on ionization, although σ_+ and σ_- are generally more useful for practical applications than σ_i.

A brief review of portions of the theory of ionization by ion impact is presented in Sec. II. General theoretical relationships among the ionization cross sections predicted for various fast-projectile ions incident on a given target gas are elaborated. In Sec. III, the problem of obtaining the simple ionization cross section from measured values of the total ion and electron production and total charge-changing cross sections is discussed. Estimates of the ionization cross sections obtained from the total ion- and electron-production measurements reported in Paper I[1] are presented in Sec. IV and compared with predictions calculated from the theory with the help of experimental values of certain atomic matrix elements deduced from previous measurements with incident protons.[6–8]

II. REVIEW OF THE THEORY OF IONIZATION BY ION IMPACT AT NONRELATIVISTIC VELOCITIES

As is well known, ionization cross sections cannot be calculated exactly even for the simplest case of protons incident on hydrogen atoms, although the wave functions for the unperturbed H atom are known completely and analytically. An infinite set of coupled differential equations would have to be solved to obtain σ_i exactly, so approximate methods must be used.

One of the most useful methods is the Born approximation,[9,10] which should be valid for high-impact velocities. The basic assumption in the Born approximation is that there is little interaction between the projectile and target. Specifically, the following assumptions are made: (1) The incident wave is undistorted by the interaction, so that the projectiles may be represented by a plane wave; (2) Excitation to any final state comes only as the result of a direct transition from the initial state, and intermediate states play no role; (3) The potential energy of the interaction between the scattered projectile and the target in its final state is small, so that the distortion of the scattered wave may be neglected. Even if these assumptions are made, an extremely difficult computation remains, at least parts of which must usually be done by numerical methods. As a consequence, the results cannot be obtained in closed analytical form, and the dependence of the results on the various parameters is not easy to ascertain.

One of the steps in ionization calculations in the full Born approximation involves the evaluation of matrix elements of the quantity $\exp(iKz)$ between the initial and final-state wave functions of the target system. Here K is the magnitude of the momentum change suffered by the projectile in the collision. Obviously, the calculation of these matrix elements requires that explicit wave functions of the states be known or assumed.

Calculations of simple ionization cross sections in the full Born approximation have been made for only a few of the simplest cases. Among these, the cases of interest here include protons and atomic hydrogen incident on atomic hydrogen,[11] protons incident on helium,[12] and He^+ ions incident on atomic hydrogen.[13] Measurements in this laboratory of the ionization cross sections for protons incident on hydrogen and helium have been compared with the theoretical calculations for these cases previously.[8] In the hydrogen case, a simple scaling procedure was used to apply the calculations for atomic hydrogen to the case of molecular hydrogen targets. This procedure allows for the fact that the ionization potentials of atomic and molecular hydrogen are different. Quite satisfactory agreement was obtained in

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both cases for proton energies above 0.5 MeV, where the Born approximation should be valid. In the present investigation, our experimental results for He^+ incident on molecular hydrogen are compared in Sec. IV with the calculations for atomic hydrogen, the same procedure as before being used to scale from the atomic to the molecular case. Good agreement is again obtained, this time for the entire energy range investigated (0.133 to 1.0 MeV).

Because the calculations in the full Born approximation are so difficult and are available for so few cases, it is useful to consider a further approximation, developed by Bethe,[9,10,14] which produces results having a simpler mathematical form. Cross sections calculated in the Bethe-Born approximation tend to the more precise results of the full Born treatment in the limit of very high impact velocities. The essential feature of the Bethe addition to the approximations consists of establishing that there is little contribution to the cross section for values of K, the projectile momentum change resulting from the collision, larger than a certain K_0 that is much less than the maximum value allowed by the conservation laws alone. In light of this fact, an integral over K that occurs in the formulation is terminated at the upper limit K_0. A factor $\exp(iKz)$ in the integrand can then be expanded, and only the first term which produces a nonvanishing contribution need be retained, for the case of very high impact velocities. With these simplifications, the general cross section for the ejection into the continuum of a single electron from the n,l shell of the target atom by a point-charge ion of charge $Z'e$ is

$$\sigma_{nl}=\frac{2\pi(Z')^2e^4c_{nl}Z_{nl}}{m_ev_0^2|E_{nl}|}\ln\frac{2m_ev_0^2}{C_{nl}}. \qquad (1)$$

Here Z_{nl} is the number of electrons in the n,l shell of the target atom, $|E_{nl}|$ is the ionization energy of this shell, m_e is the *electron* mass, e is the electron charge, and v_0 is the relative impact velocity. The quantity c_{nl} is a dipole matrix element involving the wave functions of the target atom, and C_{nl} is an energy (whose evaluation also involves a matrix element) that is of the order of $|E_{nl}|$. A further general result of the theory is that the total cross section for single electron ejection is approximately equal to σ_{nl} for the outermost shell of the target atom. As was pointed out in Sec. I, the total *apparent* ionization cross section is also approximately equal to the single-electron cross section, and therefore it should correspond at least roughly with Eq. (1).

Bethe[14] has calculated c_{nl} and C_{nl} in terms of hydrogenic wave functions and the effective nuclear charge of the target atom as seen by the electrons in the n,l shell. However, he presented explicit values only for the case of atomic hydrogen. Thus, to our knowledge, complete calculations are available in the Bethe-Born approximation for no greater a variety of cases than in the full Born treatment. Nevertheless it may be noted that Eq. (1) can be written, for a given stationary target atom, in the form

$$\sigma_i=A(Z')^2(M/E)\log_e[B(E/M)], \qquad (2)$$

where E is the kinetic energy of the incident ion, Z' its charge number, and M its mass in units of the proton mass. The constants A and B involve, aside from known constants, only the quantities c_{nl} and C_{nl}, which contain only the wave functions of the target atom. Thus the values of A and B are characteristic of the target atom and do not depend on the nature or energy of the incident ion. Therefore, an empirical evaluation of A and B for a given target atom from experimental measurements of σ_i for any one type of projectile is equivalent to an empirical evaluation of the relevant target-atom matrix elements. Given such empirical values, one may use Eq. (2) for two purposes: (1) to extrapolate the measured σ_i for the given target atom and projectile ion to energies outside the experimental range, in particular to higher energies, and (2) to estimate σ_i for the given target atom and any other projectile with a different value of Z' and/or M.

The quantities M and E appear in Eq. (2) only as the ratio E/M, so that the expression predicts that various projectiles of equal Z' but different M will have equal cross sections for equal *velocities*. This is a well known feature of the theory, which is also displayed by the results of the full Born treatment.[9–11] Even the treatment of ionization by incident electrons, while differing in some details from the theory for incident heavy ions, reduces to the same expression for very high velocities.[9,10,14] Thus it is predicted that the cross sections for electrons and for protons will be equal in a given target for sufficiently high velocities.

We have previously measured the cross sections for production of positive ions and free electrons by protons on He, Ne, Ar, H_2, N_2, O_2, and CO.[6,7] In the energy range covered (0.15 to 1.1 MeV), the charge-changing cross sections for protons in these gases are negligibly small, so that within experimental error, $\sigma_+=\sigma_-=\sigma_i$. These proton data have been fitted by a least-squares technique to Eq. (2) to obtain empirical values of A and B for all seven of the targets listed above.[8] We have also compared these proton data with electron impact data of other workers to investigate the predicted equality of the proton and electron impact cross sections. The prediction was found to be valid for proton energies above about 0.5 MeV; the equivelocity electron energy is about 270 eV.

It should be emphasized that all of the discussion of Eq. (2) above applies only to the cross sections for simple ionization events, in which the projectile ion suffers no change in its charge state. In addition, the relationships discussed here should apply, strictly speaking, only to point-charge ions, i.e., to bare nuclei. An incident ion carrying bound electrons might, how-

[14] H. A. Bethe, Ann. Phys. 5, 325 (1930).

ever, be expected to be equivalent in the simple ionization process to a partially screened point charge having an "effective" charge $Z'e$ lying somewhere between its actual net charge and its nuclear charge. The value of Z' for a given ion, and indeed the validity of the whole concept of an effective projectile charge, can for the present be evaluated only by experimental test. The concept will be useful only if Z' for a given projectile ion can be shown to be independent of the target-atom type and of the collision energy, or at least asymptotically so at high energies.

III. EVALUATION OF APPARENT IONIZATION CROSS SECTIONS

As pointed out in Sec. I, it is necessary to deduce simple ionization cross sections from the measured values of σ_+ and σ_- in order to make meaningful comparisons between experiment and theory. Unfortunately, in the case of He^+ and He^{++} projectiles, there are appreciable contributions to the total production of slow ions and electrons from charge-changing collisions in the energy range investigated, and with presently available information only an estimate can be made of the apparent cross section σ_i for simple ionization. An illustration of the nature of the difficulties may be obtained by a detailed examination of the relatively simple case of He^+ ions incident on He. Presented in Table I is

TABLE I. Possible ion-producing reactions of singly charged He^+ ions on He.

	Incident ion		Target		Fast ion		Slow ions			
(1)	He^+	+	He^0	$\rightarrow$	He^0	+	He^+			σ_{10}
(2)					He^0	+	He^{++}	+	e^-	
(3)					He^+	+	He^+	+	e^-	σ_i
(4)					He^+	+	He^{++}	+	$2e^-$	
(5)					He^{++}	+	He^0	+	e^-	σ_{12}
(6)					He^{++}	+	He^+	+	$2e^-$	
(7)					He^{++}	+	He^{++}	+	$3e^-$	
							σ_+		σ_-	

a partial listing of the more important ion-producing collisions that may occur in this case. The sum of the individual cross sections for reactions (1) and (2) is the "single-electron pickup" cross section σ_{10}; the sum for reactions (5)–(7) is the electron loss or "stripping" cross section σ_{12}; the desired apparent ionization cross section σ_i is the sum of the cross section for reaction (3) plus *twice* the cross section for reaction (4). Our measured cross sections σ_+ and σ_-, however, correspond to the total slow charge of each sign produced by all seven reactions, and must be corrected for the contributions of the charge-changing collisions to obtain σ_i. Even though data on σ_{10} and σ_{12} in this energy range are available for He^+ on several of these target gases (H_2, He, Ar, and N_2),[2,3] it is still necessary to know the relative yields of each of the separate reactions in σ_{10} and σ_{12} to make proper corrections. Unfortunately data on this point are almost nonexistent. However, it is known[15,16] that the total production of multiply charged slow ions in noble gases by He^+ ions at these energies is only a small fraction of the production of singly charged ions; thus it appears reasonable to suppose that the reactions producing slow He^{++} ions are relatively small contributors to σ_{10} and σ_{12} as well. For the present case of a helium target, it is therefore assumed that σ_{10} is mainly associated with reaction (1). For lack of other information, it is further assumed that reactions (5) and (7) make relatively minor and roughly equal contributions to σ_{12}, which is equivalent to assuming that all (1,2) collisions proceed by reaction (6). Under these assumptions

$$\sigma_i \approx \sigma_+ - (\sigma_{10} + \sigma_{12}) \tag{3}$$

and

$$\sigma_i \approx \sigma_- - 2\sigma_{12}. \tag{4}$$

The two values of σ_i thus obtained should agree, but such agreement actually reflects only agreement between our difference $(\sigma_- - \sigma_+)$ and the difference $(\sigma_{12} - \sigma_{10})$, which should hold in any event. The extent to which this last statement holds for the presently existing data is discussed in Paper I.[1]

For the higher energies above 0.6 MeV, the corrections amount at most to about 20%, so that inadequacy of the assumptions made cannot contribute an error of more than a few percent to σ_i in this region. At lower energies the situation is much less certain. However this does not really affect the goal of devising a means of extrapolating to high energies.

The other gases for which measurements of the He^+ charge-changing cross sections are available (H_2, Ar, N_2)[2,3] were treated similarly to He. In all these cases σ_{12} is at least an order of magnitude greater than σ_{10} above 0.6 MeV. If it is assumed that this is also true for the remaining cases of Ne, O_2, and CO, for which no measurements of the charge-changing cross sections are available, then our difference $(\sigma_- - \sigma_+)$ should be essentially equal to σ_{12} at these energies. Then for incident He^+ ion energies above 0.6 MeV,

$$\sigma_i \approx \sigma_+ - \sigma_{12} \approx \sigma_+ - (\sigma_- - \sigma_+) = 2\sigma_+ - \sigma_-. \tag{5}$$

Similar considerations, which will not be detailed here, are required to obtain estimates of σ_i from the measured cross sections for He^{++} projectiles.

IV. RESULTS AND DISCUSSION

The values of σ_i obtained from Eqs. (3) and (4) for He^+ ions on H_2, He, Ar, and N_2, the four target gases for which charge-changing cross sections are available,

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[16] N. V. Fedorenko and V. V. Afrosimov, Zh. Techn. Fiz. **26**, 1941 (1956) [English transl.: Soviet Phys.—Tech. Phys. **1**, 1872 (1956)].

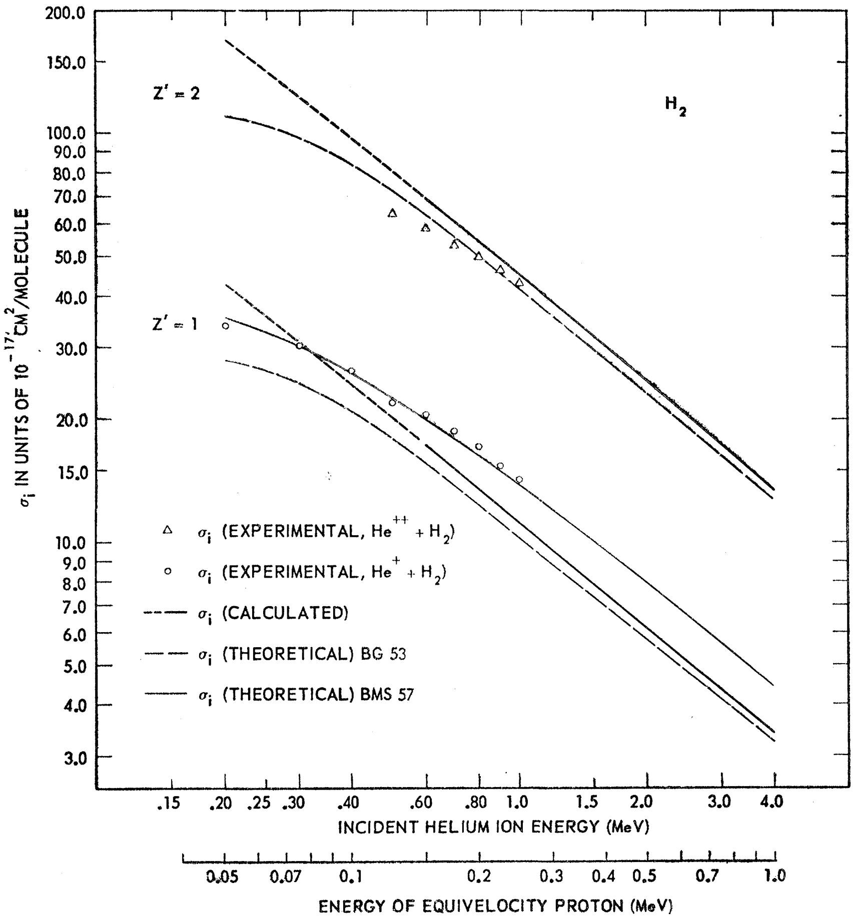

FIG. 1. Apparent ionization cross sections σ_i for helium ions incident on molecular hydrogen. Values derived from experimental measurements for incident He^+ and He^{++} ions are shown for comparison with the calculated curve of $\sigma_i=[A(Z')^2M/E]\times\ln[BE/M]$ with A and B evaluated from corresponding proton data (Ref. 8) for $Z'=1$ and $Z'=2$. Also shown for comparison are the theoretical calculations in the Born approximation for equivelocity protons on atomic hydrogen (Ref. 11) scaled to molecular hydrogen for $Z'=1$ and $Z'=2$, and those for (incident) He^+ ions on atomic hydrogen (Ref. 13) also scaled to molecular hydrogen.

are presented as the circles in Figs. 1–4; the similarly obtained values of σ_i for He^{++} ions on H_2 and He are presented as the triangular points in Figs. 1 and 2. The values obtained from Eq. (5) for He^+ ions incident on the remaining target gases, i.e., Ne, O_2, and CO, for ion energies greater than 0.6 MeV are not shown in the figures; however, their main features will be mentioned in the subsequent discussion.

The light solid curve labeled BMS 57 in Fig. 1 represents the explicit calculation in the full Born approximation for He^+ on hydrogen,[13] scaled from the atomic to the molecular target case as discussed previously. This is the only one of the present cases for which such an explicit calculation is available; it may be noted that the agreement with the present data is excellent. The lines labeled "Calculated" in each figure represent Eq. (2) for $M=4$, Z' as indicated, and the empirical values of A and B previously determined from measured proton cross sections in the same target gases. The portion of each of these curves that is drawn solid covers the energy region where the proton measurements lie (indeed this portion of the $Z'=1$ curve is equivalent, in each case, to an actual plot of the proton results referred to the "equivelocity proton energy" abscissa). The dashed portion of each "calculated" curve is extrapolation outside the data range by means of Eq. (2). The long-dash theoretical curves, labeled BG 53 in Fig. 1 and M 58 in Fig. 2, respectively, are the explicit calculations in the full Born approximation of Bates and Griffing[11] and of Mapleton,[12] respectively, for protons incident on these two targets. Both are plotted, of course, with reference to the "equivelocity proton energy" abscissa, and the BG 53 curve in Fig. 1 is scaled from the atomic to the molecular target case as usual. Each of the latter two curves is also replotted, multiplied by $4(Z'=2)$, for comparison with the He^{++} data.

It is evident from these figures that the He^{++} experimental results agree rather well with the theoretical curves, BG 53 and M 58. This agreement verifies the $(Z')^2$ dependence of the ionization cross section implied by the full Born treatment, as well as by the Bethe-Born approximation. The "calculated" cross sections, obtained from Eq. (2) by the use of empirically evaluated matrix elements, are also in agreement with the "measured" σ_i, although the agreement is not as good as in the comparison between the experimental values and the full Born calculations.

As stated earlier, the present He^+ results are in ex-

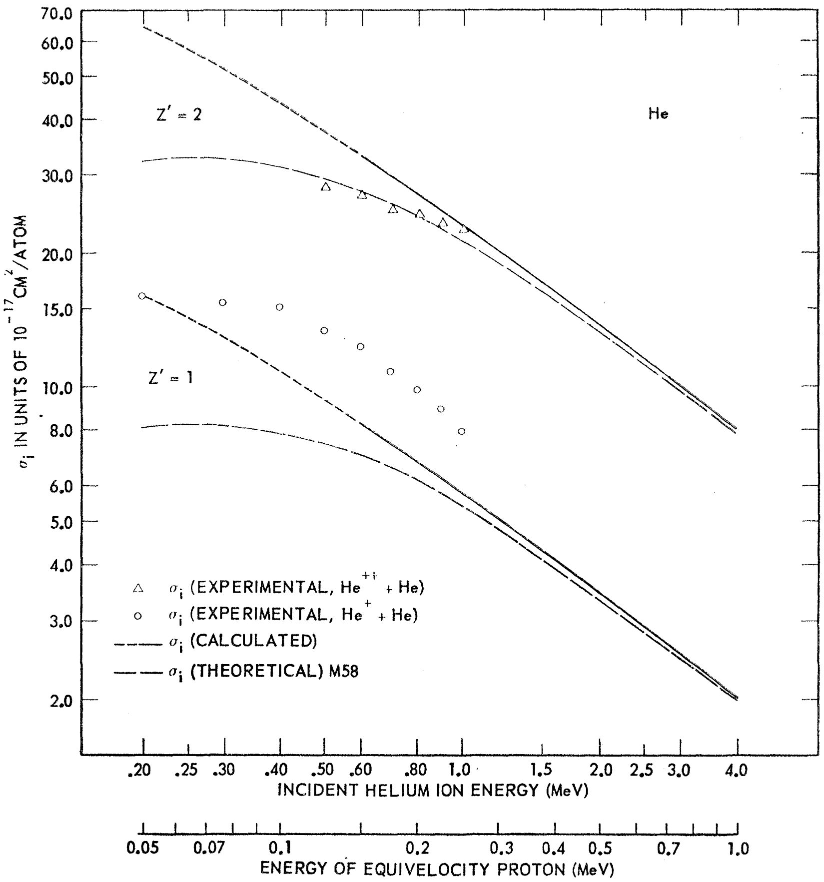

FIG. 2. Apparent ionization cross sections σ_i for helium ions incident on helium. Values derived from experimental measurements for incident He^+ and He^{++} ions are shown for comparison with the calculated curve of $\sigma_i = [A(Z')^2M/E] \ln(BE/M)$ with A and B evaluated from corresponding proton data (Ref. 8) for $Z'=1$ and $Z'=2$. Also shown for comparison are the theoretical calculations for equivelocity protons incident on helium (Ref. 12) for $Z'=1$ and $Z'=2$.

cellent agreement with the results of the explicit calculation in the full Born approximation for the one case where this calculation has been performed, that of hydrogen (BMS 57 curve in Fig. 1). In all of the figures it is seen that the present He^+ results lie between the curves calculated from Eq. (2) for $Z'=1$ and $Z'=2$. The most striking feature in all of the cases is that the results tend to run parallel to the calculated curves over all or part of the energy range covered. For the cases of H_2 and He in Figs. 1 and 2, this behavior is clearly shown only for ion energies above 0.5 MeV, but for the cases of Ar and N_2 in Figs. 3 and 4, the statement applies to essentially the entire energy range of the data. For the remaining cases of Ne, O_2, and CO, which are not shown here, only a less reliable estimate of σ_i could be obtained from Eq. (5) for energies above 0.6 MeV, because of the lack of data on the total charge-changing cross sections for these cases. However, the results for these cases also appear to run parallel to the calculated curves over this more restricted energy range. Therefore, it can be asserted that for all of these cases the results appear to have the energy dependence, at least for energies greater than 0.5 MeV, that is predicted by Eq. (2) with the empirical values of A and B obtained from incident proton measurements.

In addition, for the energy regions in which this parallel behavior is seen, the He^+ experimental results run higher than the calculated curves for $Z'=1$ by a factor of roughly 1.5 for all of the cases, including the three cases not shown in figures here. This fact implies that the factor Z' in Eq. (2) should be approximately $(1.5)^{1/2} \approx 1.2$ for all seven cases. Since the value of Z' for He^+ ions appears to be at least roughly independent of the target gas, it appears to be valid to say that He^+ can be considered to be equivalent, so far as the total probability of simple ionization collisions is concerned, and at least for energies greater than 0.5 MeV, to a point-charge ion with an effective Z' of 1.2. It is noteworthy that this value is appreciably different from the effective charge of 1.69 that is obtained in variation calculations of the ground state of the neutral helium atom. This is hardly surprising, since that is quite a different situation from the present one. However, this observation may serve as an indication that atomic wave functions that appear to be adequate to obtain good values for the energies of bound atomic states may not be adequate to produce accurate results for collision cross sections in the Born approximation.

In summary, the simple ionization cross sections for He^+ and He^{++} projectiles are reasonably well reproduced by Eq. (2), at least for energies between 0.5 and 1.0 MeV, with $Z'=1.2$ for He^+ and $Z'=2$ for He^{++}, by

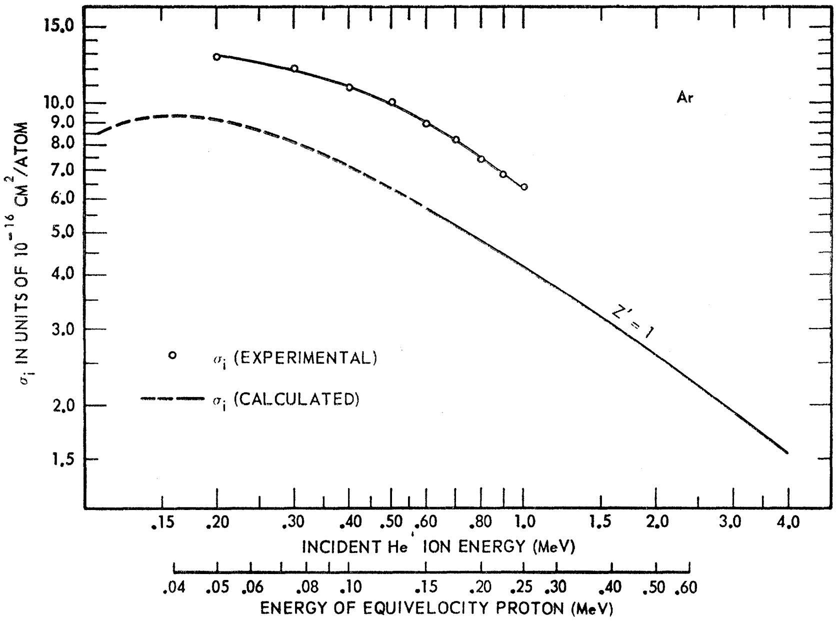

FIG. 3. Apparent ionization cross sections, σ_i, for He$^+$ ions incident on argon. Values derived from experimental measurements for incident He$^+$ ions are shown for comparison with the calculated curve of $\sigma_i = [A(Z')^2M/E] \ln(BE/M)$ with A and B evaluated from corresponding proton data (Ref. 8) for $Z'=1$.

the use of the empirical values of A and B for each target gas previously obtained from proton ionization measurements.[8] For energies less than 0.5 MeV, there is a striking contrast between the behavior observed for H_2 and He (Figs. 1 and 2) and that observed for Ar and N_2 (Figs. 3 and 4). In the former cases the correspondence between the experimental results and Eq. (2) breaks down sharply, while in the latter cases it persists throughout the energy range investigated. It is uncertain what, if anything, should be made of this observation. It is well known that even the full Born approximation tends to overestimate ionization cross sections at low energies.[10] Equation (2) represents only the asymptotic form of the Born treatment for very high velocities, and is thus not expected to be at all correct below some minimum energy. However, no very clear criteria exist for predicting the value of this minimum energy, and in the present case, it is not known whether it should be greater or less than 0.5 MeV.

Therefore, it is questionable that the values of A and B in Eq. (2) obtained for Ar and N_2 are in any sense "better" than those for H_2 and He, simply because one seems to be able to use Eq. (2) to lower energies for these cases. In our opinion the merit of this idea is

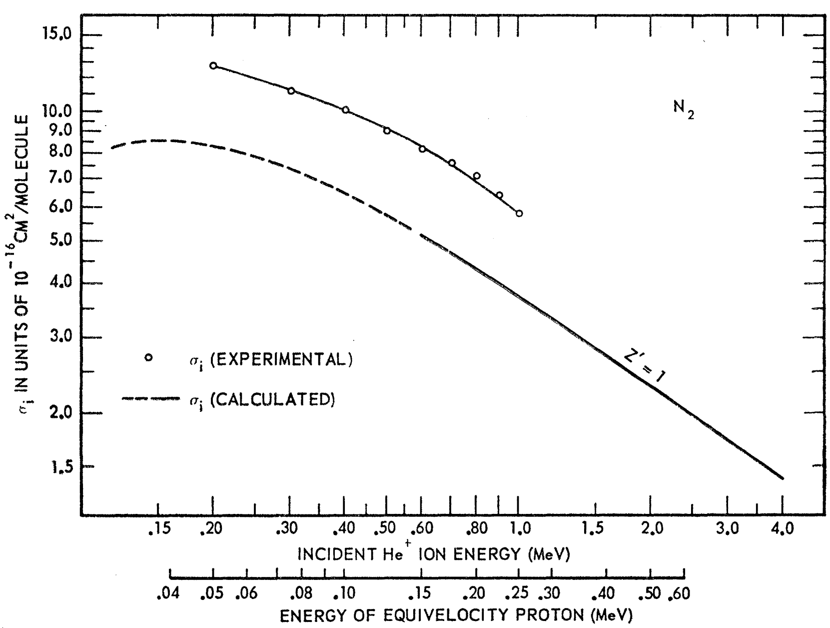

FIG. 4. Apparent ionization cross sections σ_i for He$^+$ ions incident on molecular nitrogen. Values derived from experimental measurements for incident He$^+$ ions are shown for comparison with the calculated curve of $\sigma_i = [A(Z')^2M/E] \ln(BE/M)$ with A and B evaluated from corresponding proton data (Ref. 8) for $Z'=1$.

dubious at best. The real worth of Eq. (2) lies in extrapolation of the cross sections to higher energies. Adjustment of the constants to force a fit with experimental data at energies too low for the validity of Eq. (2) would adversely affect the accuracy of this extrapolation.

Some question still remains as to just how one should obtain A and B to have the best possible values for the extrapolation. The values previously published (Table III of Ref. 8) were obtained by making a least-squares fit of Eq. (2) to all of the proton data points from 0.15 to 1.0 MeV. It is possible that this procedure may have forced the fit to lower energies than is warranted, and that the results are unduly dependent on the lowest energy data points. With the exception only of the CO results below 0.4 MeV, the proton data for all of these gases actually were found to form very nearly straight lines in a log-log plot, indicating simple E^{-C} energy dependence, with C constant. The data were in fact fitted by least squares to the function AE^{-C} as well as to Eq. (2) (Table II of Ref. 8). The usual "goodness of fit" criteria were satisfied as well or even somewhat better by these straight-line fits as by the slightly convex curves of Eq. (2).

The conclusion is that most of the experimental proton data plots do not definitely display the amount of curvature that would assure that they continue to conform to Eq. (2) to the bottom of the energy range at 0.15 MeV. As a test, new trial values of A and B were calculated according to the criterion that Eq. (2) should be made tangent to the best-fit straight lines at 1 MeV, the top end of the proton energy range, rather than made to provide the best fits to the proton data at all energies covered. The result for all cases was that the value of A was increased and that of $\ln B$ was decreased: by only some 2% for CO; by about 10% for H_2; and by roughly 20% for the other 5 cases.

With these trial values of A and B, new plots of Eq. (2) drop below the original plots (and below the data points themselves) for proton energies less than 1.0 MeV, becoming from 5 to 35% lower at 0.1 MeV. For proton energies greater than 1.0 MeV the new plots run higher than the old, but much less dramatically so. The values of σ_i calculated for 10 MeV are increased only 1% for CO, 2% for H_2, and from 8 to 10% for the other five cases; the values calculated for 100 MeV are increased 1% for CO, 3% for H_2, and from 11 to 16% for the other five cases.

It can be argued that these trial values of A and B represent extreme limits to the probable modifications of the originally tabulated values that would produce the best possible extrapolations. It is seen that the extrapolations are not greatly sensitive to modifications of this magnitude. Indeed, the changes at 10 MeV are little greater than the 6% possible systematic error originally assigned to all of the proton measurements.

Therefore, it is believed that Eq. (2) can be used, with the previously tabulated[8] values of the empirical constants A and B, to predict simple ionization cross sections for velocities up to that of a 10-MeV proton with a probable error of perhaps 10%, and up to the velocity of a 100-MeV proton with a probable error of only 15 or 20%. Relativistic effects would very likely invalidate further extrapolation in this direction. Similar extrapolation in the other direction to velocities less than that of a 0.15-MeV proton (0.6 MeV for helium ions) is not warranted.